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A gauge boson from the Kaluza-Klein approach of the Randall-Sundrum brane world

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Abstract

We clarify a mechanism to obtain a massless gauge boson from the Kaluza-Klein approach of the Randall-Sundrum(RS) brane world. This corresponds exactly to the same mechanism of achieving a localization of the gauge boson by adding both the bulk and brane mass terms. Accordingly this work puts another example for a localization-mechanism of the gauge boson on the brane.

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Recently, there have been lots of interest in the localization of 4D gravity proposed by Randall and Sundrum (RS) [1,2]. RS [2] introduced a single positive tension 3-brane and a negative bulk cosmological constant in the five-dimensional spacetime. There have been developed a large number of brane world models afterwards [3,4]. The introduction of branes usually gives rise to the "warping" of the extra dimensions, resulting in non-factorizable spacetime manifolds. More importantly, the presence of brane ($\delta(z)$-term) breaks the translational isometries in the extra dimension and requires severe boundary conditions for propagating modes . Therefore, it is a delicate issue to get a correct information for the propagation of fields in comparison with the conventional Kaluza-Klein theory.

There are approaches to confine standard particles on the brane by allowing the fields to live in the bulk spacetime. In this approach it is important to derive the zero mode effective action because its zero modes (massless modes) correspond to the standard model particles localized on the brane. For example, the zero modes of bulk scalar [5] and fermion fields can be localized on the brane Ref. [6].

On the other hand, the bulk gauge field has a different picture [7]. Its zero mode is not localized on the brane. However, more recently two interesting models appeared : One is based on the mechanism that the localization can be achieved by adding both bulk and brane mass terms[1] [8]. The other is that the localization of the bulk gauge field can be realized by taking into account the coupling between the gauge and the dilaton field [11]. Tachibana showed that these two approaches are closely related to each other [12].

In this paper, we show through the perturbation analysis of the Kaluza-Klein fields around the RS background that a mechanism that the KK gauge field possesses the U(1) gauge symmetry on the brane is a kind of the inverse Higgs mechanism.

Let us start with the RS model [2,13]

$$I = \int d^4x \int_{-\infty}^{\infty} dz \frac{\sqrt{-\hat{g}}}{16\pi G_5}(\hat{R} - 2\Lambda) - \int d^4x \sqrt{-\hat{g}_B}\sigma. \tag{1}$$

Here G_5 is the 5D Newton's constant, Λ the bulk cosmological constant of 5D spacetime, $\hat{g}_B$ the determinant of the induced metric describing the brane, and σ the tension of the brane. We consider that the value of σ is fine-tuned such that $\Lambda = -6k^2 (< 0)$ with $k = 4\pi G_5\sigma/3$. Let us introduce the perturbation around the RS background

$$ds^2 = \hat{g}_{MN}dx^M dx^N = H^{-2}(z) g_{MN} dx^M dx^N. \tag{2}$$

Here $H = k|z| + 1 (H' = k\theta(z),\ H'' = 2k\delta(z))$ is a warp-factor when one introduces a conformal coordinate z for the extra dimension instead of y. The standard Kaluza-Klein decomposition of the 5D metric perturbation[2] is given by

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[1]Authors in [8] call this the "inverse-Higgs mechanism". This is so because two bulk and boundary terms which break the gauge symmetry in the bulk are necessary to obtain the normalizable wavefunction. This broken gauge symmetry in the bulk is restored on the brane. In the previous works, it is called a sort of the "brane-Higgs effect" [9,10].

[2]Here we do not introduce the graviscalar propagation because it induces an instability problem of the RS background [14,15,10].

$$(g_{MN}) = \begin{pmatrix} \gamma_{\mu\nu} + \kappa^2 a_\mu a_\nu & -\kappa a_\mu \\ -\kappa a_\nu & 1 \end{pmatrix}, \qquad (g^{MN}) = \begin{pmatrix} \gamma^{\mu\nu} & \kappa a^\mu \\ \kappa a^\nu & (1 + \kappa^2 a \cdot a) \end{pmatrix} \tag{3}$$

with $\gamma_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, a^\mu = \eta^{\mu\nu} a_\nu$ and $a \cdot a = a^\mu a_\mu$. Here κ is the small parameter for the fluctuation analysis.

In this work, we are mainly interested in the bilinear action of the zero modes (massless modes). In general, it is a non-trivial problem to determine what the "zero mode" is if the full spacetime is not factorizable. As an ansatz for the zero mode, we assume that $h_{\mu\nu}$ and a_μ are functions of x-coordinates only : $h_{\mu\nu}(x), a_\mu(x)$. The above assumption comes from the observation that the graviton zero mode $h_{\mu\nu}$ in $\gamma_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$ depends only on "x" even if one starts from the massive approach of $\hat{h}_{\mu\nu}(x, z) = H^{3/2}\psi(z) h_{\mu\nu}(x)$ in the RS model [2]. For the zero mode solution with $m^2 = 0$, we have $\psi^0(z) = c_h H^{-3/2}$, thus we find $\hat{h}^0_{\mu\nu}(x, z) = c_h h_{\mu\nu}(x)$ with a constant c_h. For the spin-0 bulk field $\Phi(x, z) = H^{3/2}\chi(z)\phi(x)$, we have $\chi = c_\Phi H^{-3/2}$ for the zero mode and hence its localized zero mode is given by $\Phi^0(x, z) = c_\Phi \phi(x)$ [5].

Then, the five-dimensional action Eq. (1) is given by

$$\begin{aligned} I = \frac{1}{16\pi G_5} \int d^4x \sqrt{-\gamma} \Big[(R(\gamma) - \frac{\kappa^2}{4} f^2) \int dz H^{-3} + (1 + \kappa^2 a \cdot a) \int dz H^{-3} \Big(8\frac{H''}{H} - 20\frac{H'^2}{H^2} \Big) \\ + 8\kappa (\frac{1}{2} a^\mu \partial_\mu h^\alpha_\alpha + \partial_\mu a^\mu) \int dz \frac{H'}{H^4} - 2\Lambda \int dz H^{-5} \Big] \\ - \int d^4x \sqrt{-\gamma} \sqrt{|\delta^\mu_\nu + \kappa^2 a^\mu a_\nu|} \sigma \end{aligned} \tag{4}$$

with $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$. Considering the relations for the fluctuation analysis around the Minkowski space, we have

$$R(\gamma) \simeq \delta_1 R(h) + \delta_2 R(h), \tag{5}$$

$$\sqrt{|\delta^\mu_\nu + \kappa^2 a^\mu a_\nu|} \simeq 1 + \frac{1}{2}\kappa^2 a \cdot a, \tag{6}$$

$$\sqrt{-\gamma} \simeq 1 + \frac{1}{2} h^\alpha_\alpha - \frac{1}{4}(h^\beta_\alpha h^\alpha_\beta - \frac{1}{2} h^\alpha_\alpha h^\beta_\beta), \tag{7}$$

where $\delta_1 R(h)$ and $\delta_2 R(h)$ are the linear and bilinear Ricci scalar terms, respectively.

Then the bilinear action to Eq. (4) which governs the perturbative dynamics is given by

$$\begin{aligned} I_{bilinear} = \frac{\kappa^2}{16\pi G_4} \int d^4x \int dz \Big\{ -\frac{1}{4H^3} (\partial^\mu h^{\alpha\beta} \partial_\mu h_{\alpha\beta} - \partial^\mu h \partial_\mu h + 2\partial^\mu h_{\mu\nu} \partial^\nu h - 2\partial^\mu h_{\mu\alpha} \partial^\nu {h_\nu}^\alpha) \\ - \frac{1}{4H^3} (\partial_\mu a_\nu - \partial_\nu a_\mu)(\partial^\mu a^\nu - \partial^\nu a^\mu) \\ - \frac{1}{H^5} (-2k^2 + k\delta(z))(h^\beta_\alpha h^\alpha_\beta - \frac{1}{2} h^\alpha_\alpha h^\beta_\beta) + \frac{10}{H^5} (-2k^2 + k\delta(z)) a^\mu a_\mu \Big\} \end{aligned} \tag{8}$$

up to the partial integration over d^4x. Interestingly, it turns out that the terms in the last line of Eq.(8) look like the mass terms. k^2-terms come from the bulk AdS information of $\Lambda = -6k^2$, whereas $k\delta(z)$- terms arise from the presence of the brane at $z = 0$. In the case of bulk gauge boson, one inserts both the bulk and brane mass-terms by hand. Here the combined effect of the Kaluza-Klein approach with the brane world scenario makes the same thing as in the addition of the bulk and boundary mass-terms in the bulk gauge field action to obtain the normalizable wave function [8].

In order to see what physical states there are, let us analyze the field equations as below. First we wish to do it without any integration over z. From the action Eq. (8) we have the equations of motion

$$\frac{1}{H^3}\Big[\Box h_{\mu\nu} + \partial_\mu\partial_\nu h - (\partial_\mu\partial^\alpha h_{\alpha\nu} + \partial_\nu\partial^\alpha h_{\alpha\mu}) - \eta_{\mu\nu}(\Box h - \partial^\alpha\partial^\beta h_{\alpha\beta})\Big]$$
$$= \frac{4k}{H^5}(2k - \delta(z))(\frac{h}{2}\eta_{\mu\nu} - h_{\mu\nu}), \tag{9}$$
$$\Box a_\mu - \partial_\mu(\partial_\nu a^\nu) = \frac{40k}{H^5}(2k - \delta(z))a_\mu. \tag{10}$$

Here we find that the right hand terms of Eqs.(9) and (10) look like the mass-terms for graviton and graviscalar. Especially the presence of the brane at $z = 0$ gives rise to the singular behaviors and thus it requires the boundary conditions on $h_{\mu\nu}$ and a_μ along the extra direction. Hence as they stand, these are not genuine massless spin 2 and spin 1 particles. Our goal is to find massless particles.

In order to obtain a truly propagating graviton and a KK gauge boson, we have to integrate these equation over z using $\int_{-\infty}^{\infty} dz\delta(z) = 1$, $\int_{-\infty}^{\infty} dzH^{-3} = 1/k$, and $\int_{-\infty}^{\infty} dzH^{-5} = 1/2k$. Then the right hand terms of Eqs.(9) and (10) vanish identically and thus the massless modes appear as [10]

$$\Box h_{\mu\nu} + \partial_\mu\partial_\nu h - (\partial_\mu\partial^\alpha h_{\alpha\nu} + \partial_\nu\partial^\alpha h_{\alpha\mu}) - \eta_{\mu\nu}(\Box h - \partial^\alpha\partial^\beta h_{\alpha\beta}) = 0, \tag{11}$$
$$\Box a_\mu - \partial_\mu(\partial_\nu a^\nu) = 0. \tag{12}$$

This is so because the zero modes are effectively independent of the z-coordinate. In other words, in the bilinear action Eq.(8), the condition that the massless modes are localized on the brane requires the finiteness of its integral after the integration over z. In this case the mass-like terms of $a^\mu a_\mu$ and $h^\beta_\alpha h^\alpha_\beta - \frac{1}{2}h^\alpha_\alpha h^\beta_\beta$ identically disappear too. Then we can keep the U(1) gauge symmetry in the bilinear action. This is the brane-Higgs effect in our KK gauge field approach or the inverse-Higgs mechanism in the bulk gauge field approach.

In order to have the conventional form, we take the trace of Eq. (11)

$$\Box h - \partial^\alpha\partial^\beta h_{\alpha\beta} = 0. \tag{13}$$

Hence Eq. (11) becomes

$$\Box h_{\mu\nu} + \partial_\mu\partial_\nu h - \Big(\partial_\mu\partial^\alpha h_{\alpha\nu} + \partial_\nu\partial^\alpha h_{\alpha\mu}\Big) = 0. \tag{14}$$

So far we have not chosen any gauge for $h_{\mu\nu}$. Now let us choose the transverse (or harmonic) gauge in the five-dimensional spacetime. Since

$$g_{MN} = \eta_{MN} + \kappa\epsilon_{MN}, \qquad \Big(\epsilon_{MN}\Big) = \begin{pmatrix} h_{\mu\nu} & -a_\mu \\ -a_\nu & 0 \end{pmatrix}, \tag{15}$$

the five-dimensional harmonic gauge $\partial^M\epsilon_{MN} = \frac{1}{2}\partial_N\epsilon$ is equivalent to

$$\partial^\mu h_{\mu\nu} = \frac{1}{2}\partial_\nu h, \qquad\qquad \partial_\mu a^\mu = 0. \tag{16}$$

That is, the 5D harmonic gauge is split into the harmonic gauge for the 4D gravitational field and the Lorentz gauge for the 4D KK gauge field. Using these gauge conditions above, Eq. (14) and Eq. (12) become

$$\Box h_{\mu\nu} = 0, \qquad\qquad \Box a_\mu = 0, \tag{17}$$

respectively. Therefore, it proves that $h_{\mu\nu}$ and a_μ indeed represent the massless spin-2 particle (graviton) and the KK massless spin-1 particle (gauge boson) on the brane, respectively.

I. DISCUSSION

In studying the U(1) Maxwell term arisen from the 5D RS brane model, we use both the conventional Kaluza-Klein approach and the fluctuation analysis. Apparently, we find a mass-like term $k^2 a\cdot a$ from the bulk AdS space as well as a (mass-like) singular term $k\delta(z)a\cdot a$ from the presence of the brane at $z = 0$. In order to obtain the genuine massless vector, we integrate it over z. Then this term disappears in the linearized equation (or equivalently, the vanishing of the mass-like terms in the bilinear action). Previously we interpreted it as a sort of the brane-Higgs effect [9,10]: Here the isometry of extra dimension was broken spontaneously by the presence of the brane. Hence we expect that the gauge field becomes massive. However, we have found after the integration over z that the massive propagation of the KK gauge field does not reveal at the linear level (or in the bilinear action). We find the massless vector propagation. Here the procedure of the integration over z is a crucial step for obtaining the KK massless gauge boson. This actually corresponds to the procedure of obtaining localized zero modes of graviton and scalar in the bulk approach.

Finally we have a few comments in order. The first important one is that our "brane-Higgs effect" for the KK gauge field corresponds to the "inverse-Higgs mechanism" in the bulk gauge field approach of introducing two bulk and boundary mass terms [8]. Our approach is more natural than the the inverse-Higgs mechanism because our KK setting with the brane gives us the massless gauge boson, whereas in the latter case one has to introduce two mass terms by hand to obtain the massless gauge boson on the brane. But the results are the same. The second is that the procedure of obtaining the gauge boson bears a close parallel to that of the 4D massless graviton. Because the zero mode sector for the 4D graviton in the RS model was well established [2], our result for the zero mode for 4D KK gauge field is very credible. The last one concerns about the non-existence argument of the gauge boson in the Randall-Sundrum model. This is based on the the Z_2 orbifold symmetry that the vector gauge field A_μ should satisfy $A_\mu(x,-z) = -A_\mu(x,z)$ and so $A_\mu(x,0) = 0$ [1]. Hence if one requires the Z_2 orbifold symmetry in the brane world model, there will be no vector zero mode fluctuations. This is true before the integration of the bilinear action over z. However, if we accept the fact that the last condition for the presence of the gauge boson is to integrate the linearized equation (or, the bilinear action) over z, we find the massless gauge boson on the brane as an outcome of the brane-Higgs effect.

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